CALCULATION OF NITROGEN OSCILLATORY TEMPERATURE AND GAS TEMPERATURE IN AN ELECTRICAL DISCHARGE IN AIR

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UDC 537.525

An approximate analytical solution is obtained for the system of equations describing the change in oscillatory energy of nitrogen molecules and gas temperature in a glow discharge in an air flow.

The most important parameter determining efficiency of operation of electrical discharge chambers in CO_2 lasers with separate excitation of the flow by an auxiliary gas discharge in the discharge zone with subsequent addition of CO_2 in the resonator zone is the density of inversely populated nitrogen molecules. This parameter is in turn determined by the oscillatory temperature, gas temperature, and density of the medium. To calculate T_0 and T one must know the electron concentration and energy distribution, which is non-Maxwellian in a glow discharge at moderate pressures. There are major difficulties in calculating temperature in flow-type discharge chambers with a sectioned cathode system, where the spatial distribution n_e is significantly inhomogeneous. Those calculations known to the present authors were performed with the model used in [1], and have practically no relevance to real discharge chambers. The goal of the present study is to investigate the distributions of nitrogen molecule oscillatory temperature and gas temperature along a discharge chamber in the one-dimensional steady-state case.

In the equations for ε and T we will consider excitation of oscillatory levels of the fundamental electron state of the nitrogen molecule by electron shock, gas heating due to excitation of translational–rotational molecular degrees of freedom upon collision with electrons and subsequent channels of oscillatory energy relaxation:

$$N_2(v) + M \rightleftarrows N_2(v-1) + M + h\nu, \tag{1}$$

$$N_2(v) + M(k) \rightleftarrows N_2(v-1) + M(k+1) + \Delta h\nu_{\text{м}}, \tag{2}$$

where M and O_2, N_2, and H_2O molecules.

The excitation sections of oscillatory levels of the O_2 molecule fundamental electron state are small, and the process of establishing equilibrium between the various degrees of freedom of the H_2O molecules is very rapid. Therefore, it may be assumed that O_2 and H_2O are excited in equilibrium with the gas temperature. With the above assumptions the nonlinear V–V exchange equations describing the reaction of Eq. (2) reduce to linear. In moist air relatively low oscillatory temperatures are realized and oscillatorily excited nitrogen molecules are found mainly at lower levels. Therefore, the N_2 molecules will be considered as harmonic oscillators, and we will neglect nonresonant V–V exchange between N_2 molecules, which also leads to gas heating [2].

When the gas temperature in a glow discharge in air increases solely due to V–T relaxation of nitrogen molecule oscillatory energy, with the above approximation the kinetic equations for ε and T can be written in the form:

$$h\nu \frac{d(\varepsilon N_\psi)}{dt} = h\nu N_\psi n_e K_\Sigma - h\nu \frac{\varepsilon - \varepsilon_0}{(\tau P)} P N_\psi + h\nu\varepsilon \frac{dN_\psi}{dt}, \tag{3}$$

$$\frac{dT}{dt} = \frac{\gamma - 1}{\gamma} \frac{h\nu}{k} N_\psi \frac{\varepsilon - \varepsilon_0}{(\tau P)} P. \tag{4}$$

We will simplify these equations by considering conditions in a flow-type discharge chamber. In [3] the fractions of electron energy contained in the various degrees of freedom

A. N. Tupolev Aviation Institute, Kazan. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 5, pp. 808-813, May, 1984. Original article submitted March 21, 1983.

0022-0841/84/4605-0587$08.50

were calculated as functions of the parameter E/N. Using the results of [3], the first term in Eq. (3), which describes electron excitation of N_2 oscillations, can be represented approximately in the form $\alpha W(x)$, where we take α = const. In Eqs. (3), (4) the quantity ε_0 is much less than ε. Analysis of numerical calculation results shows that in the temperature range T = 290-500°K neglect of ε_0 has practically no effect on the accuracy of ε and T calculations. In this temperature range, at pressures characteristic of flow-type reaction chambers the quantity $\beta = (\gamma - 1)/\gamma$ can also be considered constant. Moreover, we will neglect the electron gas pressure in comparison to the neutral gas pressure, i.e., we take P = kTN. Transition to the new variable dx = vdt in Eqs. (3), (4), simplification, and transformation with consideration of the equations of state and continuity

$$\rho v = \text{const}$$

yields

$$-\frac{v_0 P_0}{T_0}\frac{d}{dx}\left(\frac{E_\sigma T}{P}\right) = \alpha W(x) - \frac{E_\ominus P}{(\tau P)}, \tag{5}$$

$$\frac{v_0 P_0}{T_0}\frac{dT}{dx} = \beta\frac{E_\ominus P}{(\tau P)}. \tag{6}$$

From Eqs. (5), (6) with initial conditions $E_\ominus(0) = 0$ and $T(0) = T_0$ we find

$$\beta\frac{E_\ominus T}{P} + T - T_0 = \frac{\alpha\beta T_0}{v_0 P_0}\int_0^x W(x)\,dx. \tag{7}$$

We can obtain a nonlinear differential equation for the gas temperature from Eqs. (6), (7)

$$(\tau P)\,T\frac{dT}{dx} + \frac{T_0 P^2(x)}{v_0 P_0}(T - T_0) = \frac{\alpha\beta T_0^2}{v_0^2 P_0^2}P^2(x)\int_0^x W(x)\,dx. \tag{8}$$

The dependence of (τP) on T is quite complex [4], and therefore Eq. (8) cannot be solved analytically. However, over the temperature range 290-500°K to an accuracy of 3% the dependence of (τP) on T can be represented in the form

$$\tau P = \frac{1}{T[a + b(T - T_0)]}, \tag{9}$$

where a and b are constants. Substitution of Eq. (9) in Eq. (8) leads to the differential equation

$$\frac{T_x'}{a + b(T - T_0)} + \frac{T_0 P^2(x)}{v_0 P_0}(T - T_0) = \frac{\alpha\beta T_0^2}{v_0^2 P_0^2}P^2(x)\int_0^x W(x)\,dx$$

with known solution

$$\Delta T = T - T_0 = \frac{a}{b}\left[\frac{\Omega_1(x)}{1 + \frac{aT_0}{v_0 P_0}\int_0^x P^2(x)\,\Omega_1(x)\,dx} - 1\right], \tag{10}$$

where

$$\Omega_1(x) = \exp\left\{\frac{aT_0}{v_0 P_0}\int_0^x P^2(x)\,dx + \frac{\alpha\beta b T_0^2}{v_0^2 P_0^2}\int_0^x P^2(x)\int_0^x W(y)\,dy dx\right\}.$$

The values of $E_\ominus$ are defined by Eq. (7). Calculation of ΔT by Eq. (10) yields results differing from a numerical solution of Eqs. (5), (6) with no simplification of the dependence of (τP) on T by not more than 0.3%.

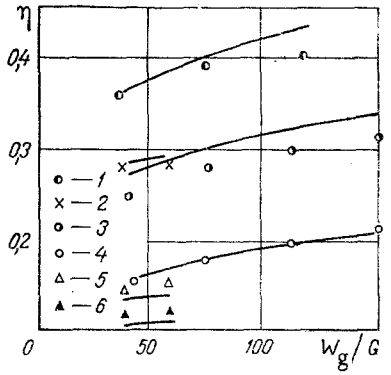

Fig. 1. Comparison of calculation results with experimental data of [6]. Curves, calculation; points, experimental η values. 1) $X_{H_2O} = 1.8 \cdot 10^{-2}$, G = 17.8 g·sec^{-2}; 2) $1.8 \cdot 10^{-2}$, 35.9; 3, 4) $1.5 \cdot 10^{-2}$, 17.8; 5, 6) $1.5 \cdot 10^{-2}$, 35.9; 4, 6) η values at end of working zone; 1, 2, 3, 5) at distance of 162 mm from end. W_g/G, J/g.

We will now consider further the relaxation zone $x \geqslant x_a$, where W(x) = 0. The quantities

$$E_0(x_a) = E_0^1, \quad T(x_a) = T_1, \quad v(x_a) = v_1, \quad P(x_a) = P_1,$$

which can be calculated with the expressions presented above, will be considered known. From Eqs. (5), (6) at W(x) = 0 with consideration of initial conditions in the section x_α we find

$$\beta \frac{E_0 T}{P} + T - T_1 = \beta \frac{E_0^1 T_1}{P_1}. \tag{11}$$

Using Eqs. (6), (9), and (11), we find a differential equation for T, integration of which gives

$$\Delta T = T - T_1 = \frac{a^*}{b} \left\{ \frac{\Omega_2(x)}{1 + \dfrac{a^*}{a^* + \dfrac{\beta b E_0^1 T_1}{P_1}} [\Omega_2(x) - 1]} - 1 \right\}. \tag{12}$$

Here

$$\Omega_2(x) = \exp\left[\frac{T_1}{v_1 P_1} \left(a^* + \frac{\beta b E_0^1 T_1}{P_1} \right) \int_{x_a}^{x} P^2(x)\, dx \right],$$

$$a^* = b(T_1 - T_0) + a.$$

The E_0 values are defined by Eqs. (11), (12).

As is evident from the expressions obtained, to calculate E_0 and T it is necessary to know the power density distribution W(x) along the chamber. According to the data of [5] W(x) can be approximated by the expression

$$W(x) = W_0(x) \left[1 + c \sin\left(2\pi \frac{x}{x_0} \right) \right], \tag{13}$$

where c = const (0 < c < 1). Calculations of E_0 and T were performed using Eq. (13) and compared to results of calculating E_0 and T at $W(x) = W_0(x)$. Values and distributions of W(x) characteristic of real reaction chambers were used. Analysis of the results reveals that if $x \geqslant 5x_0$, then the values of E_0 and T calculated at $W(x) = W_0(x)$ differ from E_0 and T calculated with Eq. (13) by not more than 3%. When $x \geqslant 10x_0$, periodic inhomogeneities in power-density distribution in the discharge zone may be neglected and mean values used in the calculations.

Use of the expressions obtained to calculate E_0 and T in the discharge zone is difficult because of their complex form. Therefore, to simplify we will consider Eq. (10) with the condition

$$\varphi^2(x)=\left\{\frac{\alpha\varkappa bT_0^2}{v_0^2P_0^2}\int_0^x P^2(x)\int_0^x W(y)\,dydx\right\}\ll 1.$$

Then after transforming Eq. (10) we obtain

$$\Delta T=\frac{\alpha\varkappa T_0}{v_0P_0}\left\{\int_0^x W(x)\,dx-\frac{\int_0^x W(x)\exp\left[\frac{aT_0}{v_0P_0}\int_0^x P^2(y)\,dy\right]dx}{\exp\left[\frac{aT_0}{v_0P_0}\int_0^x P^2(y)\,dy\right]}\right\}. \tag{14}$$

The solution of the original system of equations at $(\tau P)^{-1}=aT$ is a special case of Eq. (14). Calculations with Eq. (14) give lowered T values. By comparing results of T calculations with Eqs. (10) and (14), it was established that with use of a values determined from the approximate equation $a=[T(\tau P)]^{-1}$ at T = 343°K, the gas temperature values found from Eq. (14) differ from numerical solutions of the original system with no simplification by not more than ±3%.

In the general case, it is necessary to consider the increase in gas temperature due to excitation of translational–rotational degrees of freedom upon collisions with electrons. As follows from [3], an amount of power $(1-\alpha)W(x)$ is expended in this fashion. Then the system of kinetic equations for E_0T/P will have the form

$$\frac{v_0P_0}{T_0}\frac{d}{dx}\left(\frac{E_0T}{P}\right)=\alpha W(x)-\frac{E_0P}{(\tau P)}, \tag{15}$$

$$\frac{v_0P_0}{T_0}\frac{dT}{dx}=\beta\frac{E_0P}{(\tau P)}+\beta(1-\alpha)W(x). \tag{16}$$

When (τP) is approximated by Eq. (9), this system of equations has no analytical solution. However, as an approximation we may assume that the "slow" heating due to V–T relaxation of nitrogen molecule oscillatory energy and "rapid" heating due to direct contribution of energy to translational and rotational degrees of freedom are not related to each other, i.e., that the temperature increase is determined by the sum of the two processes. In this approximation the solution of Eqs. (15), (16) will have the form

$$\beta\frac{E_0T}{P}+T-T_0=\frac{\beta T_0}{v_0P_0}\int_0^x W(x)\,dx, \tag{17}$$

$$\Delta T=T-T_0=\frac{T_0}{v_0P_0}\beta(1-\alpha)\int_0^x W(x)\,dx+K\frac{a}{b}\left[\frac{\Omega_1(x)}{1+\frac{aT_0}{v_0P_0}\int_0^x P^2(x)\,\Omega_1(x)\,dx}-1\right]. \tag{18}$$

Introduction of the coefficient K was necessary because at K = 1 calculations of T by Eq. (18) give lowered values. Analysis of results obtained indicates that at a value K = 1.15 calculation of T by Eq. (18) differs from results of numerical solution of the original system by not more than 1%.

In [6] the fraction of electrical energy producing gas heating was measured. Using specified external discharge parameters presented in [6], the expressions obtained herein may be used to calculate the dependence of η on power supplied. The solid curves of Fig. 1 are the results of such calculations, while the points are experimental η values from [6]. As is evident, experiment and calculation agree completely satisfactorily.

Thus, the expressions obtained above may be used for engineering calculations of discharge chambers operating with atmospheric air.

NOTATION

n_e, electron concentration; ε, mean number of oscillatory quanta per nitrogen molecule; ε_0, equilibrium value of ε; E_0, oscillatory energy of nitrogen molecules per unit volume; N, number of molecules per unit volume; N_ψ, number of nitrogen molecules per unit volume; T, gas temperature; T_0, nitrogen molecule oscillatory temperature; P, pressure; v, gas velocity; (τP), normalized oscillatory-translational relaxation time for nitrogen molecule oscillatory energy in air; t, time; x, coordinate along discharge chamber; x_0, distance between adjacent cathodes along discharge chamber; W, power density distribution along chamber; W_g, total power contained in gas; K_Σ, net rate of excitation of oscillatory levels of nitrogen molecule ground state oscillatory levels (per electron) defined by electron energy distribution and oscillatory temperature T_0; $h\nu$, energy of nitrogen molecule oscillatory quantum; v_0, v value at x = 0; P_0, P value at x = 0; T_0, T value at x = 0; γ, adiabatic index; k, Boltzmann's constant; η, fraction of electrical energy expended in gas heating; G, air flow rate; X_{H_2O}, partial pressure of water vapor.

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EXPERIMENTAL INVESTIGATION OF THE INTERACTION BETWEEN THE PLASMA OF AN ELECTRICALLY EXPLODED WIRE AND A PLANE OBSTACLE

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UDC 533.924

The results of an investigation of the physical processes that occur when pulses of dense low-temperature plasma, generated by electrical explosion of a wire, flow over a plane dielectric obstacle are presented.

The interaction of dense pulsed flows of low-temperature plasma with the surface of a solid is an important subject for scientific investigation. The impact drag and flow of a pulsed plasma beam around different obstacles have been investigated in [1-6], where the nature of the radiation was studied, and the temperature and density of the gas discharge and the erosion plasma in the layer perturbed by the obstacle were estimated. However, there appear to be no data available in the literature on the relationship between the properties of a collision-compressed plasma and the nature of its interaction with the surface of the obstacle. In this paper we study this relationship using the example of the collisional slowing down of a plasma from an electrically exploded wire. The electrical-explosion method was chosen to generate the plasma because it enables one to obtain dense, high-

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 5, pp. 813-819, May, 1984. Original article submitted December 22, 1982.

0022-0841/84/4605-0591$08.50